Short Papers

A Method for Evaluating the Noise Temperature of Microwave Thermal Noise Sources by Introducing an Auxiliary Transmission Line

YOSHIHIKO KATO, KOJI KOMIYAMA, AND ICHIRO YOKOSHIMA

Abstract —**A new method is proposed to evaluate the noise temperature of a microwave thermal noise source. The noise temperature correction dominated by the transmission line between the termination and the output is expressed in terms of directly measurable parameters. It can be determined by introducing an auxiliary transmission line composed of two lines connected back-to-back, each of which is identical to that of the unknown source.**

I. Introduction

A microwave thermal noise source is composed of a matched resistive termination maintained at a fixed temperature and a transmission line to transmit the thermal noise generated in the termination to its output port. The noise temperature at the output port is a function of the physical temperature of the termination and the temperature and distributed transmission loss along the transmission line. Therefore, the determination of the noise temperature contribution due to the transmission line is required to measure the equivalent noise temperature at the output port.

Up to this time, two methods to evaluate the noise temperature of a thermal noise source have been reported: one is to calculate the noise temperature contribution of the transmission line at the outpout by using the physical temperatures measured at several points along the uniform transmission line and the losses at the corresponding points estimated from the insertion loss measurements of the identical line at the several separate temperatures [1]. The other is to directly measure the temperature correction to be added to the termination temperature without any measurement of the temperature and loss distribution through a procedure to replace the matched termination by a sliding short [2]. Both methods require complicated measurements and error evaluations.

With the view to reducing these problems, a new evaluation method has been proposed, introducing an auxiliary transmission line composed of a pair of transmission lines connected back-to-back, each of which is identical to that of the unknown noise source.

II. Measurement Principle

The measurement principle will be explained by referring to Fig. 1. The noise temperature of the unknown noise source (T_x) is expressed as

$$T_x = \alpha T_G + T_{e1} \tag{1}$$

where α is the ratio of the available power at the output of the

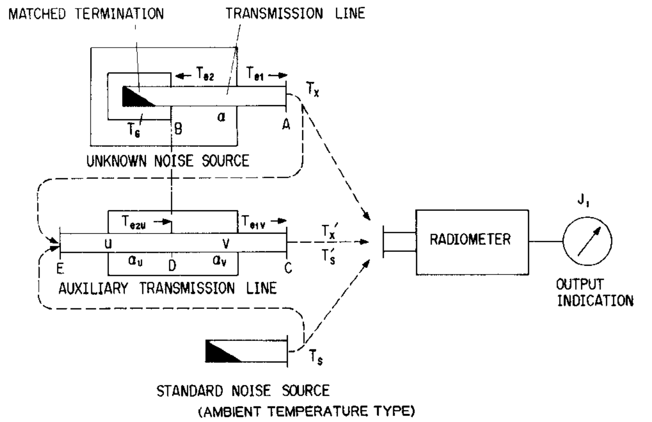

Fig. 1. An unknown noise source and a noise temperature measurement system using the auxiliary transmission line: the unknown noise source is composed of a matched termination of physical temperature T_G and a transmission line (AB); the auxiliary transmission line is composed of a pair of transmission lines connected back-to-back, each of which is identical to that of the unknown, and the standard noise source is operated at ambient temperature.

transmission line (AB) to that at the input, and T_{e1} is the noise temperature contribution at the output port A of the noise generated in the two-port AB. Equation (1) can be rewritten as

$$T_x = T_{m1} + T_G \tag{2a}$$

$$T_{m1} = T_{e1} - (1-\alpha) T_G \tag{2b}$$

where T_{m1} is the noise temperature correction to be determined.

In order to demonstrate the principle of this method, the following approximations are assumed:

1) $T_{e1} = T_{e2} = T_{e2u} = T_{e1v}$
2) $\alpha = \alpha_u = \alpha_v$
3) $|\Gamma_x| = |\Gamma_x'| = |\Gamma_s| = 0$

where the subscripts u and v denote the sections ED and DC of the auxiliary line, respectively, and Γ_x, Γ_x', and Γ_s are the reflection coefficients for the unknown noise source, the auxiliary line with the unknown source, and the standard noise source, respectively.

We introduce an actually measurable parameter with the radiometer expressed as

$$R = \frac{J_x' - J_x}{J_s - J_x} \tag{3}$$

where J_x and J_s are the radiometer output indications for the unknown and the standard source, respectively, and J_x' is for the auxiliary line with the unknown source.

Then, T_{m1} is expressed by (see Appendix I)

$$T_{m1} = \frac{R(T_s - T_G)}{\alpha^2 + \alpha + R}. \tag{4}$$

Meanwhile, α is determined by measuring the transmission loss of the auxiliary line, but it can also be measured from (see Appendix I)

$$\alpha = \sqrt{\frac{J_s' - J_x'}{J_s - J_x}} \tag{5}$$

Manuscript received March 21, 1987, revised August 3, 1987
The authors are with the Electrotechnical Laboratory, Radio Electronics Section, Ibaraki-ken, 305 Japan.
IEEE Log Number 8717597

0018-9480/88/0100-0145$01.00 ©1988 IEEE

where J_s' is the radiometer output indication for the auxiliary line with the standard noise source. Thus, T_{m1} is expressed in terms of actually measurable parameters.

Here, it is stressed that the standard noise source should be operated at ambient temperature since its noise temperature is determined from the physical temperature of its termination.

III. Considerations on Errors

The followings are considered as principal errors in determination of T_{m1}: errors due to approximations 1 to 3, errors due to the measurement of R and α, errors due to supplementary connection lines to be used for performing this measurement, etc.

1) Errors Due to Approximations 1 and 2: The parameter T_e depends not only on the distributed temperatures and circuit parameters of the transmission line but also on the reflection coefficient of the termination connected at the input port [2], and α depends on them as well [5]. Therefore, the approximations do not hold in general and their effect on error can not be ignored. However $T_{e1v} = T_{e1}$ and $T_{e2u} = T_{e2}$ may be realized if those transmission lines are produced with sufficient care.

Then, we will investigate the effect of the difference between T_{e1} and T_{e2}, and the differences of α_u and α_v to α.

So let

$$T_{e2} = T_{e1}(1+\delta) \tag{6}$$

$$\alpha_u = \alpha(1+\epsilon_u) \tag{7}$$

$$\alpha_v = \alpha(1+\epsilon_v) \tag{8}$$

where $|\delta|$, $|\epsilon_u|$, and $|\epsilon_v| \ll 1$ in general.

The maximum error for T_{m1} due to ignoring these is obtained from the difference between T_{m1} derived using (6)–(8) and that obtained in the former section:

$$|\Delta T_{m1}| \simeq \frac{\alpha^2(|\epsilon_u| + |\epsilon_v|) + \alpha(|\epsilon_u| + |\delta|)}{\alpha^2 + \alpha + R} T_{m1}. \tag{9}$$

The maximum value of $|\delta|$ is (see Appendix II)

$$|\delta| \simeq \frac{1-\alpha}{\alpha}. \tag{10}$$

The value increases with the decrease of α, i.e., the increase of the loss.

It is difficult to determine α_u and α_v separately but $\alpha_u\alpha_v$ can be measured by loss measurement. Then, $|\epsilon_u|$ and $|\epsilon_v|$ will be approximately estimated by

$$|\epsilon_u| = |\epsilon_v| \simeq \frac{|\sqrt{\alpha_u\alpha_v} - \alpha|}{\alpha}. \tag{11}$$

For instance, if $\alpha = 0.05$ dB ($|\delta|$ is less than 0.012) and $|\epsilon_u| = |\epsilon_v| = 0.01$ dB for $T_s = 296$ K, $T_G = 78$ K, $T_{m1} = 2$ K, and $R = 0.018$, $|\Delta T_{m1}|$ is nearly equal to 0.02 K.

2) Error Due to Approximation 3: The difference of R's when the reflection coefficient of each noise source is ignored and is taken into count is approximately expressed by

$$|\Delta R| = \frac{|R-1||T_s - T_r||\Gamma_s|^2 + |T_s' - T_r||\Gamma_s'|^2 + |R||T_s - T_r||\Gamma_s|^2}{|T_s - T_s|} \tag{12}$$

where T_r is the effective noise temperature when looking into the radiometer at the input (see Appendix I).

T_{m1} can be evaluated by substituting this $|\Delta R|$ into (13), to be mentioned in the next paragraph.

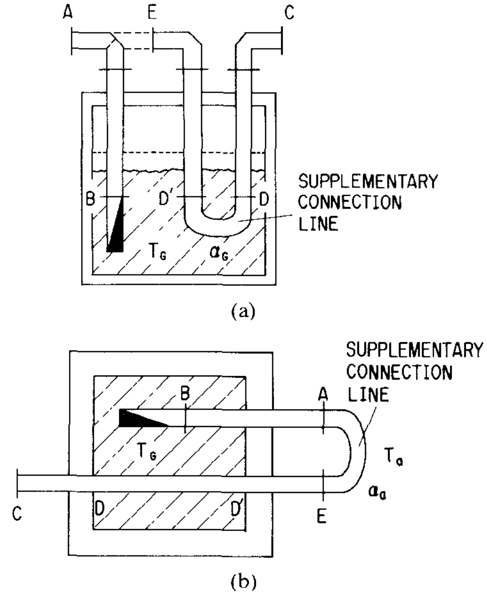

Fig. 2. An unknown noise sources fabricated together with the auxiliary transmission line. (a) Vertical type (b) Horizontal type

If $|\Gamma_s| = |\Gamma_s'| = 0.02$ for $T_s = T_r = 296$ K, $T_G = 78$ K, and $R \ll 1$, $|\Delta T_{m1}| = 0.09$ K. Therefore, the magnitudes of the reflection coefficients must be as small as possible so that they may not become significant errors to T_{m1}.

3) Errors Due to the Measurements of R and α: The contributions of the measurement errors of R and α to $|\Delta T_{m1}|$ are, respectively,

$$|\Delta T_{m1}|_R \simeq \frac{\alpha^2 + \alpha}{(\alpha^2 + \alpha + R)^2}|T_s - T_G||\Delta R| \tag{13}$$

and

$$|\Delta T_{m1}|_\alpha \simeq \frac{(2\alpha+1)R}{(\alpha^2 + \alpha + R)^2}|T_s - T_G||\Delta\alpha|. \tag{14}$$

$|\Delta R|$ is dominated by the accuracy of the attenuator built in the radiometer and the fluctuation of its output.

4) Errors due to the Supplementary Connection Lines: In practice, the auxiliary line may be fabricated as shown in Fig. 2, different from Fig. 1; i.e., it will be installed with the unknown noise source in the same vessel, such as liquid-nitrogen-cooled noise sources.

When considering the losses of a line (AE) to connect the unknown noise source and the auxiliary line, and a line (DD′) to connect two transmission lines (u and v) of the auxiliary line, the T_{m1}'s are written, respectively, as

$$T_{m1} = \frac{R(T_s - T_G)}{(\alpha^2 + \alpha)\alpha_G + R} \tag{15}$$

and

$$T_{m1} = \frac{R(T_s - T_G) - \alpha^2(1-\alpha_a)(T_a - T_G)}{\alpha^2\alpha_a + \alpha + R}. \tag{16}$$

In the case where both losses are ignored, errors for T_{m1} are, respectively,

$$|\Delta T_{m1}|_{\alpha_G} \simeq \frac{(\alpha^2 + \alpha)(1-\alpha_G)}{\alpha^2 + \alpha + R} T_{m1} \tag{17}$$

and

$$|\Delta T_{m1}|_{\alpha_a} \approx \frac{\alpha^2(1-\alpha_a)}{\alpha^2+\alpha+R}(T_a - T_G). \tag{18}$$

where α_G is the available power ratio of the line (DD′) in the cold section, and T_a and α_a are, respectively, the temperature and the available power ratio of the line (AE) in the ambient temperature section.

5) Others: This method may cause more serious error for noise sources with coaxial transmission lines than waveguides because the following problems are added: insertion loss and its reproducibility, the identity of support and temperature-controlling beads for center conductor, and the interaction of the higher order modes near conductors. Therefore, their effect must be estimated quantitatively.

IV. Conclusions

A new method has been presented to evaluate the noise temperature correction of the transmission line of a microwave thermal noise source from only directly measurable parameters by introducing an auxiliary transmission line. This method has the merit that the noise temperature can be evaluated without knowing the thermal profile and electrical loss of its transmission line, but its usefulness should be further investigated by the comparison of uncertainties between this method and others.

Appendix I
Derivation of T_{m1} and α

When the unknown source is connected to port E of the auxiliary line, the noise temperature at port C can be written as

$$T_x' = \alpha^2 T_x + (\alpha+1)T_{e1}. \tag{A1}$$

With the use of (1) to (2), T_x' can be expressed approximately as

$$T_x' = (\alpha^2+\alpha+1)T_{m1} + T_G. \tag{A2}$$

When a noise source of T_i is connected to a tuned total power type radiometer, the output indication is given by [3]

$$J_i = G\left[(1-|\Gamma_{ir}|^2)T_i + |\Gamma_{ii}|^2 T_r + 2\,Re\{\Gamma_{ii}T_c\} + T_n\right] \tag{A3a}$$

$$\Gamma_{ii} = \frac{\Gamma_i - \Gamma_r^*}{1-\Gamma_i\Gamma_r}. \tag{A3b}$$

Equation (A3a) is rewritten by

$$J_i = G\left[(1-|\Gamma_{ii}|^2)(T_i - T_r) + T_k\right] \tag{A4a}$$

$$T_k = T_r + 2\,Re\{\Gamma_{ii}T_c\} + T_n \tag{A4b}$$

where Γ_i and Γ_r are the reflection coefficients of the noise source and the radiometer, respectively; G is the conversion coefficient of noise temperature to output indication, and T_r, T_k and T_c are, respectively, the effective temperature when looking into the radiometer at the input, the effective noise temperature at the input by noise wave generating in the circuit and going toward the detector, and the correlation temperature of the radiometer. Here T_r and T_c can be measured by a method but the correlation factor can also be made negligibly small [4].

If $|\Gamma_s|$, $|\Gamma_x'|$, and $|\Gamma_x|$ are equal to zero and $|\Gamma_r|$ is tuned out, (3) becomes, by using (2a) and (A2),

$$R = \frac{T_s' - T_x'}{T_s - T_x} = \frac{(\alpha^2+\alpha)T_{m1}}{T_s - T_G - T_{m1}}, \tag{A5}$$

By solving the equation for T_{m1}, (4) is obtained.

Meanwhile, α is determined by performing the following procedure: The standard noise source is connected to the auxiliary line at the port E. Then the noise temperature at the port C is

$$T_s' = \alpha^2 T_s + (\alpha+1)T_{e1}. \tag{A6}$$

By using (A1) and (A6), α can be expressed as

$$\alpha = \sqrt{\frac{T_s' - T_x'}{T_s - T_x}}. \tag{A7}$$

Then, (5) is obtained.

Appendix II
Derivation of Evaluation Expression of the Difference between T_{e1} and T_{e2}

The transmission line is assumed to be uniform. The line is divided into n segments. Let the physical temperature and the available power ratio in the ith segment be T_i and α_i, respectively. Then, T_{e1} and T_{e2} are, respectively,

$$T_{e1} = \sum_{i=1}^{n-1} T_i(1-\alpha_i)\prod_{j=i+1}^{n}\alpha_j + T_n(1-\alpha_n) \tag{A8}$$

and

$$T_{e2} = T_1(1-\alpha_1) + \sum_{i=2}^{n} T_i(1-\alpha_i)\prod_{j=1}^{i-1}\alpha_j. \tag{A9}$$

By using the condition $0<\alpha_i<1$, and (A8) and (A9), the minimum and maximum values of T_{e2}/T_{e1} are obtained:

$$\frac{T_{e2}}{T_e} = \frac{\prod_{j=1}^{n}\alpha_j\left\{T_1(1-\alpha_1)\left(\prod_{j=1}^{n}\alpha_j\right)^{-1} + \sum_{i=2}^{n}T_i(1-\alpha_i)\left(\prod_{j=i}^{n}\alpha_j\right)^{-1}\right\}}{\sum_{i=1}^{n-1}T_i(1-\alpha_i)\prod_{j=i+1}^{n}\alpha_j + T_n(1-\alpha_n)}$$

$$> \frac{\prod_{j=1}^{n}\alpha_j\sum_{i=1}^{n}T_i(1-\alpha_i)}{\sum_{i=1}^{n}T_i(1-\alpha_i)} = \prod_{j=1}^{n}\alpha_j = \alpha \tag{A10}$$

and

$$\frac{T_{e2}}{T_{e1}} = \frac{T_1(1-\alpha_1) + \sum_{i=2}^{n}T_i(1-\alpha_i)\prod_{j=1}^{i-1}\alpha_j}{\prod_{j=1}^{n}\alpha_j\left\{\sum_{i=1}^{n-1}T_i(1-\alpha_i)\left(\prod_{j=1}^{i}\alpha_j\right)^{-1} + T_n(1-\alpha_n)\left(\prod_{j=1}^{n}\alpha_j\right)^{-1}\right\}}$$

$$< \frac{\sum_{i=1}^{n}T_i(1-\alpha_i)}{\prod_{j=1}^{n}\alpha_j\sum_{i=1}^{n}T_i(1-\alpha_i)} = \frac{1}{\prod_{j=1}^{n}\alpha_j} = \frac{1}{\alpha}. \tag{A11}$$

Then, (10) is obtained approximately.

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